

# Neutrino Masses from Cosmological Probes

Øystein Elgarøy<sup>†</sup> and Ofer Lahav<sup>‡</sup>

<sup>†</sup> Institute of theoretical astrophysics, University of Oslo, P.O. Box 1029, N-0315 Oslo, Norway

<sup>‡</sup> Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, UK

E-mail: oelgaroy@astro.uio.no; lahav@star.ucl.ac.uk

## Abstract.

There is a renewed interest in constraining the sum of the masses of the three neutrino flavours by using cosmological measurements. Solar, atmospheric, reactor and accelerator neutrino experiments have confirmed neutrino oscillations, implying that neutrinos have non-zero mass, but without pinning down their absolute masses. While it is established that the effect of light neutrinos on the evolution of cosmic structure is small, the upper limits derived from large-scale structure could help significantly to constrain the absolute scale of the neutrino masses. It is also important to know the sum of neutrino masses as it is degenerate with the values of other cosmological parameters, e.g. the amplitude of fluctuations and the primordial spectral index. A summary of cosmological neutrino mass limits is given. Current results from cosmology set an upper limit on the sum of the neutrino masses of  $\sim 1$  eV, somewhat depending on the data sets used in the analyses and assumed priors on cosmological parameters. It is important to emphasize that the total neutrino mass ('hot dark matter') is derived assuming that the other components in the universe are baryons, cold dark matter and dark energy. We assess the impact of neutrino masses on the matter power spectrum, the cosmic microwave background, peculiar velocities and gravitational lensing. We also discuss future methods to improve the mass upper limits by an order of magnitude.

PACS numbers: 95.35.+d, 14.60.Pq, 98.62.Py, 98.80.Es

## 1. Introduction

The connection between neutrino masses and cosmic structure formation was realised in the 1970's by Zeldovich and others, but for a long time cosmologists were mostly interested in neutrino masses in the  $\sim 10$  eV range, i.e. massive enough to make up all of the dark matter. The downfall of the top-down 'hot dark matter' scenario of structure formation, and the fact that no evidence for neutrino masses existed before Super-Kamiokande detected oscillations of atmospheric neutrinos in 1998, explains why there was very little continuous interest in this sub-field. However, the detection of neutrino oscillations showed that neutrinos indeed have a mass, i.e. 'hot dark matter' does exist, even if only in a very small amount.

The wealth of new data from the cosmic microwave background (CMB) and large-scale structure (LSS) in the last few years indicate that we live in a flat Universe where  $\sim 70$  % of the mass-energy density is in the form of dark energy, with matter making up the remaining 30 % . The WMAP data combined with other large-scale structure data [1, 2] give impressive support to this picture. Furthermore, the baryons contribute only a fraction  $f_b = \Omega_b/\Omega_m \sim 0.15$  ( $\Omega_b$  and  $\Omega_m$  are, respectively, the contribution of baryons and of all matter to the total density in units of the critical density  $\rho_c = 3H_0^2/8\pi G = 1.879 \times 10^{-29} h^2 \text{ g cm}^{-3}$ , where  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$  is the present value of the Hubble parameter) of this, so that most of the matter is dark. The exact nature of the dark matter in the Universe is still unknown. Relic neutrinos are abundant in the Universe, and from the observations of oscillations of solar and atmospheric neutrinos, as well as in the reactor-based KamLAND experiment and in the accelerator-based long-baseline K2K experiment, we know that neutrinos have a mass [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and will make up a fraction of the dark matter. The KamLAND collaboration has also reported evidence for spectral distortion in the  $\bar{\nu}_e$  spectrum [13], further strengthening the case for neutrino oscillations and allowing rather strict bounds on the the neutrino oscillation parameters to be found. However, the oscillation experiments can only measure differences in the squared masses of the neutrinos, and not the absolute mass scale, so they cannot, at least not without extra assumptions, tell us how much of the dark matter is in neutrinos. From general arguments on structure formation in the Universe we know that most of the dark matter has to be cold, i.e. non-relativistic when it decoupled from the thermal background. Neutrinos with masses on the eV scale or below will be a hot component of the dark matter. If they were the dominant dark-matter component, structure in the Universe would have formed first at large scales, and smaller structures would form by fragmentation (the 'top-down' scenario). However, the combined observational and theoretical knowledge about large-scale structure gives strong evidence for the 'bottom-up' picture of structure formation, i.e. structure formed first at small scales. Hence, neutrinos cannot make up all of the dark matter (see e.g. [14] for a review). Neutrino experiments give some constraints on how much of the dark matter can be in the form of neutrinos. Studies of the energy spectrum in tritium decay [15] provide an upper limit

on the effective electron neutrino mass involved in this process of 2.2 eV (95 % confidence limit). For the effective neutrino mass scale involved in neutrino less double beta decay a range 0.1-0.9 eV has been inferred from the claimed detection of this process [16, 17]. If confirmed, this result would not only show that neutrinos are Majorana particles (i.e. their own antiparticles), but also that the neutrino masses are in a range where they are potentially detectable with cosmological probes.

The structure of this review is as follows. Sections 2 and 3 describe the role of massive neutrinos in structure formation and for the CMB anisotropies, respectively. In section 4 we give an overview of recent cosmological neutrino mass limits. Section 5 discusses other methods for constraining neutrino masses. We discuss challenges for the future in section 6.

## 2. Massive neutrinos and structure formation

The relic abundance of neutrinos in the Universe today is straightforwardly found from the fact that they continue to follow the Fermi-Dirac distribution after freeze-out, and their temperature is related to the CMB temperature  $T_{\text{CMB}}$  today by  $T_\nu = (4/11)^{1/3}T_{\text{CMB}}$ , giving

$$n_\nu = \frac{6\zeta(3)}{11\pi^2}T_{\text{CMB}}^3, \quad (1)$$

where  $\zeta(3) \approx 1.202$ , which gives  $n_\nu \approx 112 \text{ cm}^{-3}$  at present. By now, massive neutrinos will have become non-relativistic, so that their present contribution to the mass density can be found by multiplying  $n_\nu$  with the total mass of the neutrinos  $m_{\nu,\text{tot}} \equiv \sum m_\nu$ , giving

$$\Omega_\nu h^2 = \frac{m_{\nu,\text{tot}}}{94 \text{ eV}}, \quad (2)$$

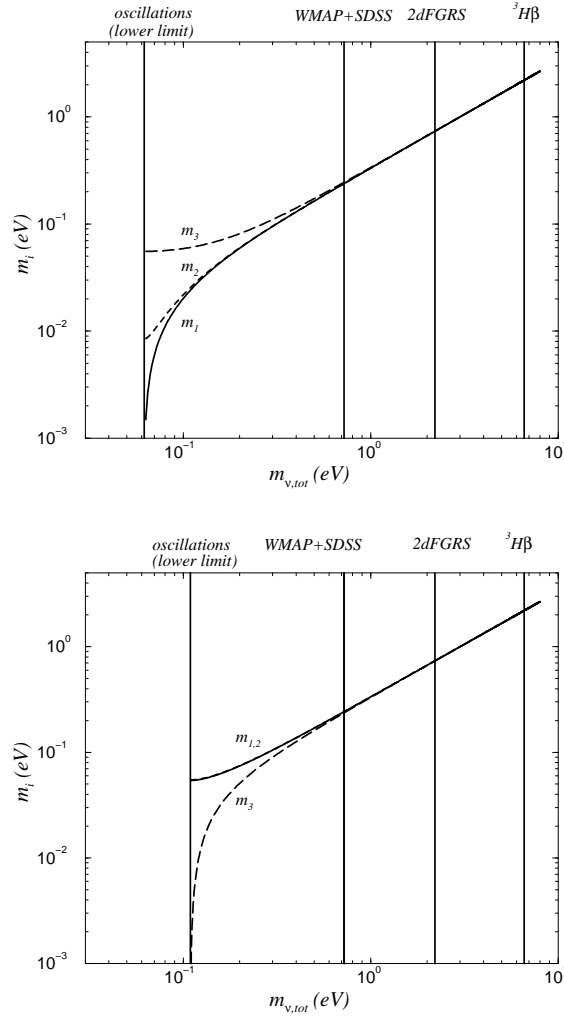
for  $T_{\text{CMB}} = 2.726 \text{ K}$ . Several effects could modify this simple relation. If any of the neutrino chemical potentials were initially non-zero, or there were a sizable neutrino-antineutrino asymmetry, this would increase the energy density in neutrinos and give an additional contribution to the relativistic energy density. However, from Big Bang Nucleosynthesis (BBN) one gets a very tight limit on the electron neutrino chemical potential, since the electron neutrino is directly involved in the processes that set the neutron-to-proton ratio. Also, within the standard three-neutrino framework one can extend this limit to the other flavours as well. The results of the KamLAND experiment [18] confirmed the Large Mixing Angle (LMA) solution for the solar neutrino oscillations, and combined with the atmospheric data indicating maximal mixing in this sector, it has been shown that flavour equilibrium is established between all three neutrino species before the epoch of BBN [19, 20, 21, 22], so that the BBN constraint on the electron neutrino asymmetry applies to all flavours, which in turn implies that the lepton asymmetry cannot be large enough to give a significant contribution to the relativistic energy density. One should note, however, that these bounds can be evaded if there is an additional source of energy density to compensate the  $\nu_e$  asymmetry [23]. Analyses

of WMAP and 2dFGRS data give independent, although not quite as strong, evidence for small lepton asymmetries [24, 25]. Within the standard picture, equation (1) should be accurate, and therefore any constraint on the cosmic mass density of neutrinos should translate straightforwardly into a constraint on the total neutrino mass, according to equation (2). If a fourth, light ‘sterile’ neutrino exists, sterile-active oscillations would modify this conclusion, and could also have implications for e.g. BBN [26]. See [27] for a review. No sterile neutrinos are required to explain the solar and atmospheric neutrino oscillation data [28], and the only hint so far comes from the possible detection of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations with a small mixing angle and a mass-square difference  $\sim 1 \text{ eV}^2$  at the Liquid Scintillating Neutrino Detector (LSND) [29]. Since there are only two independent mass-squared differences in the standard three-neutrino scenario, and they are orders of magnitude smaller, this hints at the existence of a fourth, light sterile neutrino. However, this has found to be highly disfavoured by the solar and atmospheric neutrino oscillation data [30, 31]. The status of the LSND results will in the near future be clarified by the MiniBooNE experiment [32].

In a recent paper Beacom, Bell & Dodson [33] pointed out that if neutrinos couple to a light scalar field, they might annihilate as they become non-relativistic. If this were the case, massive neutrinos would have a negligible effect on the matter power spectrum. However, Hannestad [34] has pointed out that such a scenario would have a marked effect on the CMB power spectrum and is strongly disfavoured by the current data. We will therefore assume in the following that no such non-standard couplings of neutrinos exist.

Finally, we assume that the neutrinos are nearly degenerate in mass. Current cosmological observations are sensitive to neutrino masses  $\sim 1 \text{ eV}$  or greater. Since the mass-square differences are small, the assumption of a degenerate mass hierarchy is therefore justified. This is illustrated in figure 1, where we have plotted the mass eigenvalues  $m_1, m_2, m_3$  as functions of  $m_{\nu, \text{tot}} = m_1 + m_2 + m_3$  for  $\Delta m_{21}^2 = 7 \times 10^{-5} \text{ eV}^2$  (solar) and  $\Delta m_{32}^2 = 3 \times 10^{-3} \text{ eV}^2$  (atmospheric), for the cases of a normal hierarchy ( $m_1 < m_2 < m_3$ ), and an inverted hierarchy ( $m_3 < m_1 < m_2$ ). As seen in the figure, for  $m_{\nu, \text{tot}} > 0.4 \text{ eV}$  the mass eigenvalues are essentially degenerate.

Here we look at cosmological models with four components: baryons, cold dark matter, massive neutrinos, and a cosmological constant. Furthermore, we restrict ourselves to adiabatic, linear perturbations. The basic physics is then fairly simple. A perturbation mode of a given wavelength  $\lambda$  can grow if it is greater than the Jeans wavelength  $\lambda_J$  determined by the balance of gravitation and pressure, or rms velocity in the case of massless particles. Above the Jeans scale, perturbations grow at the same rate independently of the scale. Long after matter-radiation equality, all interesting scales are above  $\lambda_J$  and grow at the same rate, and in models where all the dark matter is cold, the time and scale dependence of the power spectrum can therefore be separated at low redshifts. Light, massive neutrinos can, however, move unhindered out of regions below a certain limiting length scale, and will therefore tend to damp a density perturbation at a rate which depends on their rms velocity. The presence of massive neutrinos therefore



**Figure 1.** Neutrino mass eigenvalues as functions of  $m_{\nu, \text{tot}}$  for the cases of normal (top panel) and inverted (bottom panel) hierarchies. The vertical line marked ‘oscillations’ is the lower limit derived from the measured mass-squared differences. The vertical line marked ‘WMAP+SDSS’ is the recent limit derived in [55] from WMAP+SDSS, the vertical line marked ‘2dFGRS’ is the limit from the 2dFGRS derived in [51], and the line marked ‘ $^3\text{H}\beta$ ’ is the upper limit from Tritium  $\beta$  decay.

introduces a new length scale, given by the size of the co-moving Jeans length when the neutrinos became non-relativistic. In terms of the comoving wavenumber, this is given by

$$k_{\text{nr}} = 0.026 \left( \frac{m_\nu}{1 \text{ eV}} \right)^{1/2} \Omega_{\text{m}}^{1/2} h \text{ Mpc}^{-1}, \quad (3)$$

for three equal-mass neutrinos, each with mass  $m_\nu$ . The growth of Fourier modes with  $k > k_{\text{nr}}$  will be suppressed because of neutrino free-streaming. The free-streaming scale varies with the cosmological epoch, and the scale and time dependence of the power spectrum cannot be separated, in contrast to the situation for models with cold dark

matter only.

The transfer functions of the perturbations in the various components provide a convenient way of describing their evolution on different scales. Using the redshift  $z$  to measure time, the transfer function is formally defined as

$$T(k, z) = \frac{\delta(k, z)}{\delta(k, z = z_*)D(z_*)} \quad (4)$$

where  $\delta(k, z)$  is the density perturbation with wavenumber  $k$  at redshift  $z$ , and  $D$  is the linear growth factor. The normalization redshift  $z_*$  corresponds to a time long before the scales of interest have entered the horizon. The transfer function thus gives the amplitude of a given mode  $k$  at redshift  $z$  relative to its initial value, and is normalized so that  $T(k = 0, z) = 1$ . The power spectrum of the matter fluctuations can be written as

$$P_m(k, z) = P_*(k)T^2(k, z), \quad (5)$$

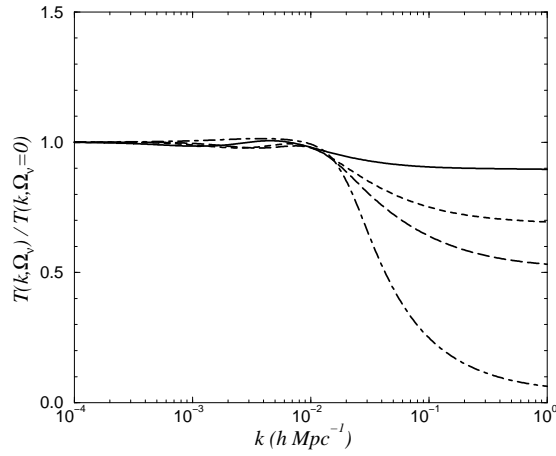
where  $P_*(k)$  is the primordial spectrum of matter fluctuations, commonly assumed to be a simple power law  $P_*(k) = Ak^n$ , where  $A$  is the amplitude and the spectral index  $n$  is close to 1. It is also common to define power spectra for each component, see [35] for a discussion. Note that the transfer functions and power spectra are insensitive to the value of the cosmological constant as long as it does not shift the epoch of matter-radiation equality significantly.

The transfer function is found by solving the coupled fluid and Boltzmann equations for the various components. This can be done using one of the publicly available codes, e.g. CMBFAST [36] or CAMB [37]. In figure 2 we show the transfer functions for models with  $\Omega_m = 0.3$ ,  $\Omega_b = 0.04$ ,  $h = 0.7$  held constant, but with varying neutrino mass per flavour  $m_\nu \equiv m_{\nu, \text{tot}}/3$ . One can clearly see that the small-scale suppression of power becomes more pronounced as the neutrino fraction  $f_\nu \equiv \Omega_\nu/\Omega_m$  increases.

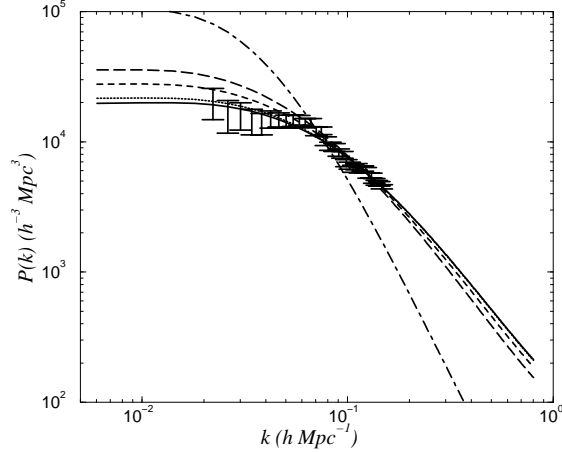
The effect is also seen in the power spectrum, as shown in figure 3. Note that the power spectra shown in the figure have been convolved with the 2dFGRS window function, as described in [38]. Furthermore, we have taken the possible bias of the distribution of galaxies with respect to that of the dark matter into account by leaving the overall amplitude of each power spectrum as a free parameter to be fitted to the 2dFGRS power spectrum data (the vertical bars in the figure). For a discussion of bias in the context of neutrino mass limits, see [39]. Because the errors on the data points are smaller at small scales, these points are given most weight in the fitting, and hence the power spectra in the figure actually deviate more and more from each other on *large* scales as  $m_\nu$  increases. One can see from the figure that a neutrino mass of  $m_\nu = 0.5$  eV or larger is in conflict with the data. The suppression of the power spectrum on small scales is roughly proportional to  $f_\nu$ :

$$\frac{\Delta P_m(k)}{P_m(k)} \approx -8f_\nu. \quad (6)$$

This result can be understood qualitatively from the equation of linear growth of density perturbations and the fact that only a fraction  $(1 - f_\nu)$  of the matter can cluster when



**Figure 2.** Ratio of the transfer functions (at  $z = 0$ ) for various values of  $\Omega_\nu$  to the one for  $\Omega_\nu = 0$ . The other parameters are fixed at  $\Omega_m = 0.3$ ,  $\Omega_b = 0.04$ ,  $h = 0.7$ . The solid line is for neutrino mass per flavour  $m_\nu = 0.1$  eV, the dashed line is for  $m_\nu = 0.3$  eV, the long-dashed line is for  $m_\nu = 0.5$  eV, and the dot-dashed line corresponds to  $m_\nu = 2$  eV.



**Figure 3.** Power spectra for neutrino mass per flavour  $m_\nu = 0$  (full line),  $m_\nu = 0.1$  (dotted line),  $m_\nu = 0.3$  (dashed line),  $m_\nu = 0.5$  (long-dashed line), and  $m_\nu = 3$  eV (dot-dashed line). The other parameters are fixed at  $\Omega_m = 0.3$ ,  $\Omega_b = 0.04$ ,  $h = 0.7$ . The vertical bars are the 2dFGRS power spectrum data points.

massive neutrinos are present [40].

### 3. Constraints from the CMB alone

Neutrino masses also give rise to effects in the CMB power spectrum. If their masses are smaller than the temperature at recombination their effect is very similar to that of massless neutrinos [41]. For slightly larger masses, there is an enhancement of the acoustic peaks with respect to the massless case, as shown in figures 4 and 5. In figure 5 we show two cases, one (top) where we fix  $\Omega_m = \Omega_b + \Omega_c + \Omega_\nu = 0.3$ ,  $\Omega_b = 0.04$ ,  $\Omega_\Lambda = 0.7$   $h = 0.7$ . At the second (bottom) we fix  $\Omega_c = 0.26$ ,  $\Omega_b = 0.04$ ,  $h = 0.7$ , while keeping a flat universe  $\Omega_b + \Omega_c + \Omega_\nu + \Omega_\Lambda = 1.0$ . the purpose of this comparison is to check if the change caused by adding massive neutrinos is not just due to the decrease in the amount of cold dark matter. Indeed we see that qualitatively the effect is the same in both cases and even somewhat larger when the amount of cold dark matter is held fixed. Note that this case the position of the peaks will change slightly with  $\Omega_\nu$  since varying it now also implies changing  $\Omega_\Lambda$ , and hence the angular diameter distance to the last scattering surface. This effect is not obvious in the figure, since we plot the ratios of the angular power spectra.

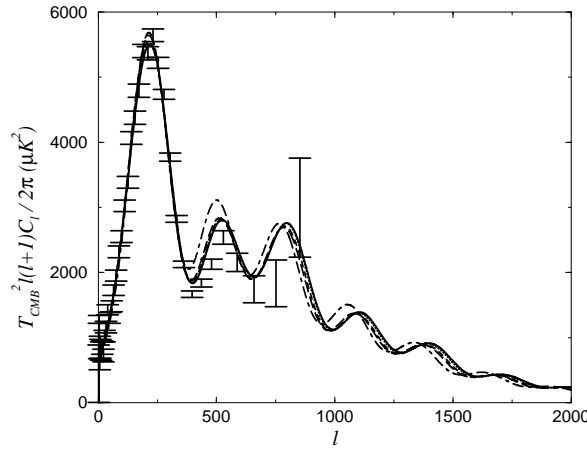
From figure 4 it looks like the WMAP data alone should be sufficient to provide an upper limit on the neutrino masses. For example in the case of fixed total  $\Omega_m = 0.3$  we find that  $\Delta\chi^2$  for the model with  $m_\nu = 0.3$  compared with the reference model with  $m_\nu = 0$  is larger than 40. However, note that all other parameters have been fixed in figures 4 and 5, and that there are severe degeneracies between  $m_\nu$  and other parameters like  $n$  and  $\Omega_b h^2$ . There are severe degeneracies between  $m_\nu$  and other parameters like  $n$  and  $\Omega_b h^2$ . The full analysis of the WMAP data alone in [44] gave no upper limit on  $m_\nu$ . On the other hand [42] have claimed an upper limit of 2.0 eV from CMB alone, but emphasizing that one cannot improve the limit below 1.5 eV. The differences in the conclusions from various studies in the literature on neutrino masses from CMB alone might be due to the assumed priors over other cosmological parameters.

Analytic considerations in [42] provide insight into the effect of the neutrinos on the CMB. A key point is to consider the redshift when neutrino mass becomes non-relativistic,  $1+z_{nr} = 6.24 \times 10^4 \Omega_\nu h^2$ . For a recombination redshift  $z_{rec} = 1088$  this means that neutrinos became non-relativistic before recombination  $z_{nr} > z_{rec}$  if  $\Omega_\nu h^2 > 0.017$  (i.e. a total neutrino mass  $m_{\nu, \text{tot}} > 1.6$  eV). In other words, if they are heavier than the above value, they act as matter and if neutrinos are very light they nearly behave like radiation. Indeed as shown in [42] the dependence of the position of the first peak and the normalized height of the first peak on  $\Omega_\nu h^2$  has a turning point at  $\Omega_\nu h^2 \approx 0.017$ . This value also affects CMB anisotropy via the modification of the integrated Sachs-Wolfe effect due to the massive neutrinos.

Future CMB missions like Planck ‡ will provide high-resolution maps of the CMB temperature and polarization anisotropies. Gravitational lensing of these maps causes distortions, and Kaplinghat, Knox & Song [43] have shown that this effect can be used to obtain very stringent limits on neutrino masses from the CMB alone. For Planck,

‡ <http://www.esa.int/science/planck>





**Figure 4.** CMB power spectra for neutrino mass per flavour  $m_\nu = 0$  (full line),  $m_\nu = 0.1$  (dotted line),  $m_\nu = 0.3$  (dashed line),  $m_\nu = 0.5$  (long-dashed line), and  $m_\nu = 3$  eV (dot-dashed line). The other parameters are fixed at  $\Omega_m = 0.3$ ,  $\Omega_b = 0.04$ ,  $h = 0.7$ . The vertical bars are the WMAP power spectrum data points

they predict a sensitivity down to 0.15 eV, whereas a future experiment with higher resolution and sensitivity can possibly reach the lower bound  $\sim 0.06$  eV set by the neutrino oscillation experiments.

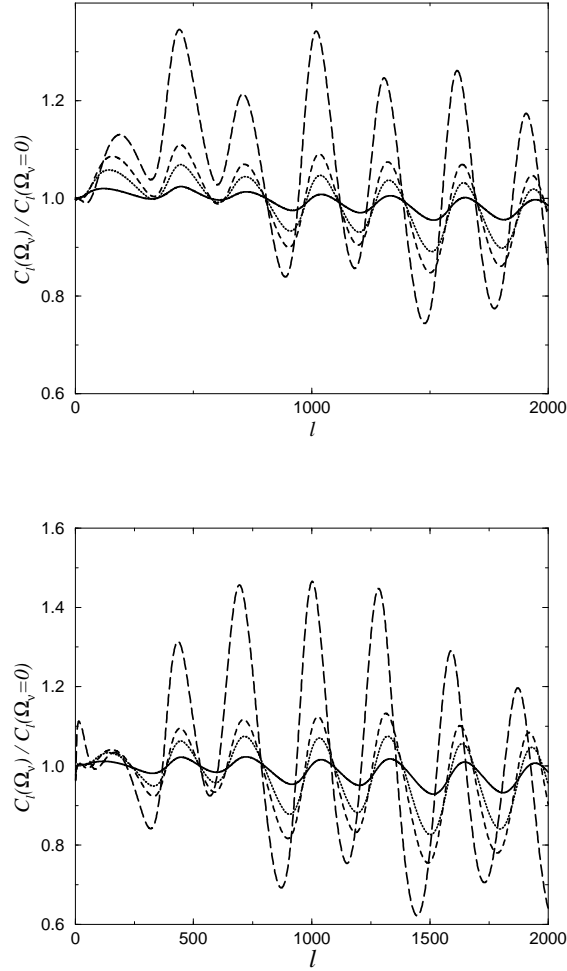
#### 4. Recent cosmological neutrino mass limits

In an important paper Hu, Eisenstein & Tegmark [45] showed that one could obtain useful upper limits on neutrino masses from a galaxy redshift survey of the size and quality of the Sloan Digital Sky Survey (SDSS). Based on a Fisher matrix analysis, their prediction was that the SDSS should be able to obtain a  $2\sigma$  detection of  $N$  nearly degenerate massive neutrino species with mass

$$m_\nu \geq 0.65 \left( \frac{\Omega_m h^2}{0.1N} \right)^{0.8} \text{ eV}, \quad (7)$$

which for  $N = 3$ ,  $\Omega_m h^2 = 0.135$  predicts that a 95 % confidence upper limit  $m_{\nu, \text{tot}} \leq 1.03$  eV should be obtainable.

As far as we are aware of, the first cosmological neutrino mass limit after the detection of neutrino oscillations was derived by Croft, Hu & Davé [46]. Their main piece of data on large-scale structure was the matter power spectrum derived from the Lyman- $\alpha$  forest. Combining with two other measurements of the amplitude of matter fluctuations, the COBE normalization and  $\sigma_8$ , they obtained a 95% confidence upper limit of  $m_{\nu, \text{tot}} < 16.5$  eV, see table 4. Shortly thereafter, Fukugita, Liu & Sugiyama [47] derived a stronger upper limit of 2.7 eV by combining the COBE normalization of the matter power spectrum with constraints on  $\sigma_8$  from cluster abundances, using strong



**Figure 5.** *Top* : The ratio of CMB angular power spectra for various values of  $\Omega_\nu$  to the one for  $\Omega_\nu = 0$ . The other parameters are fixed at  $\Omega_m = \Omega_b + \Omega_c + \Omega_\nu = 0.3$ ,  $\Omega_b = 0.04$ ,  $\Omega_\Lambda = 0.7$   $h = 0.7$ . The full line is for neutrino mass per flavour  $m_\nu = 0.1$  eV, the dotted line is for  $m_\nu = 0.3$  eV, the dashed line is for  $m_\nu = 0.5$  eV, and the long-dashed line corresponds to  $m_\nu = 2$  eV. *Bottom* : The ratio of CMB angular power spectra for various values of  $\Omega_\nu$  to the one for  $\Omega_\nu = 0$ . The other parameters are fixed at  $\Omega_c = 0.26$ ,  $\Omega_b = 0.04$ ,  $h = 0.7$ , while keeping a flat universe  $\Omega_b + \Omega_c + \Omega_\nu + \Omega_\Lambda = 1.0$ . The lines have the same meaning as in the figure above.

priors  $\Omega_m < 0.4$ ,  $h < 0.8$ , and  $n = 1.0$ . The first limit derived from a combined analysis of CMB and large-scale structure data was that of Wang, Tegmark & Zaldarriaga [49]. They included the power spectrum of galaxies derived from the PSCz survey [50] and obtained an upper limit of 4.2 eV. Going down table 4 one notes a marked improvement in the constraints after the 2dFGRS power spectrum became available. After WMAP, there is a further tendency towards stronger upper limits, reflecting the dual role of the CMB and large-scale structure in constraining neutrino masses: the matter power spectrum is most sensitive to the ratio  $\Omega_\nu/\Omega_m$ , but one needs good constraints on

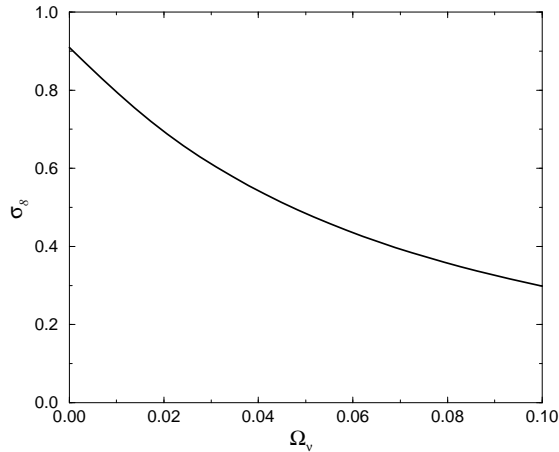
Reference	CMB	LSS	Other data	$m_{\nu,\text{tot}}$ upper limit
[46]	—	Ly $\alpha$	COBE norm., $h = 0.72 \pm 0.08$ , $\sigma_8 = 0.56\Omega_m^{0.47}$	16.5 eV
[47]	—	$\sigma_8$	$\Omega_m < 0.4$ , $\Omega_b h^2 = 0.015$ , $h < 0.8$ , $n = 1.0$	2.7 eV
[49]	pre-WMAP	PSCz, Ly $\alpha$	—	4.2 eV
[51]	None	2dFGRS	BBN, SNIa, HST, $n = 1.0 \pm 0.1$	2.2 eV
[52]	pre-WMAP	2dFGRS	—	2.5 eV
[53]	pre-WMAP	2dFGRS	SNIa, BBN	0.9 eV
[1]	WMAP+CBI+ACBAR	2dFGRS	Ly $\alpha$	0.71 eV
[24]	WMAP+Wang comp.	2dFGRS	HST, SNIa	1.01 eV
[48]	WMAP+CBI+ACBAR	2dFGRS	X-ray	$0.56^{+0.30}_{-0.26}$ eV
[44]	WMAP	SDSS	—	1.7 eV
[54]	WMAP	2dFGRS+SDSS	—	0.75 eV
[41]	WMAP+ACBAR	2dFGRS+SDSS	—	1.0 eV
[55]	WMAP	SDSS	bias	0.54 eV
[47]	WMAP alone	—	—	2.0 eV
[56]	WMAP	SDSS + Ly $\alpha$	—	0.42 eV

the other relevant cosmological parameters to break degeneracies in order to obtain low upper mass limits. The limit will depend on the datasets and priors used in the analysis, but it seems like we are now converging to the precision envisaged in [45]. In [55], galaxy-galaxy lensing was utilized to extract information about the linear bias parameter in the SDSS, making a direct association between the galaxy and matter power spectra, and hence resulting in a stronger constraint on the neutrino mass than would have been possible using just the shape of the galaxy power spectrum. Finally, in [56] Lyman- $\alpha$  forest constraints were added to the data sets in [55], resulting in the very strong limit  $m_{\nu,\text{tot}} < 0.42$  eV at 95 % confidence.

## 5. Other cosmological probes of neutrino masses

### 5.1. The clustering amplitude

Direct probes of the total matter distribution avoid the issue of bias and are therefore ideally suited for providing limits on the neutrino masses. Several ideas for how this can be done exist. In [47] the normalization of the matter power spectrum on large scales derived from COBE was combined with constraints on  $\sigma_8$  from cluster abundances and



**Figure 6.** The clustering amplitude  $\sigma_8$  as a function of  $\Omega_\nu$  for models with amplitude fitted to the WMAP data. Other parameters are fixed at ‘concordance’ values.

a constraint  $m_{\nu,\text{tot}} < 2.7$  eV obtained, although with a fairly restricted parameter space. However,  $\sigma_8$  is probably one of the most debated numbers in cosmology at the moment [57], and a better understanding of systematic uncertainties connected with the various methods for extracting it from observations is needed before this method can provide useful constraints. The potential of this method to push the value of the mass limit down also depends on the actual value of  $\sigma_8$ : the higher  $\sigma_8$  turns out to be, the less room there will be for massive neutrinos. As an illustration we show in figure 6 the value of  $\sigma_8$  as a function of varying  $\Omega_\nu$  with the remaining cosmological parameters fixed at their ‘concordance’ values. For a given value of  $m_\nu$ , one fits the corresponding CMB power spectrum to the data. This in turn leads to a best-fit amplitude and a prediction for  $\sigma_8$  for the given value of  $m_\nu$ . If one then has an independent measurement of  $\sigma_8$ , one can infer the value of  $m_\nu$ . In figure 6 the amplitude of the power spectrum has been fixed by fitting to the WMAP data. The claimed detection of a non-zero neutrino mass in [48] can be seen to be due to the use of the cluster X-ray luminosity function to constrain  $\sigma_8$ , giving  $\sigma_8 = 0.69 \pm 0.04$  for  $\Omega_m = 0.3$  [58]. If a value of  $\sigma_8$  at the higher end of the results reported in the literature is used instead, e.g.  $\sigma_8 = 0.9$  for  $\Omega_m = 0.3$  from [59], one gets a very tight upper limit on  $m_\nu$ , but no detection of  $m_\nu > 0$ . It is clearly important that systematic issues related to the various methods of obtaining  $\sigma_8$  are settled. The evolution of cluster abundance with redshift may provide further constraints on neutrino masses [60].

### 5.2. The Lyman- $\alpha$ forest

Simulations support the picture that the matter density field is locally related to the optical depth for absorption of light emitted by quasars by the Lyman- $\alpha$  clouds. Hence the Lyman- $\alpha$  forest provides constraints on the matter power spectrum on scales of

$k \sim 1 \, h \text{Mpc}^{-1}$ , where the effect of massive neutrinos is most visible. It was used in [46] to derive a limit  $m_{\nu, \text{tot}} < 16.5 \text{ eV}$ . However, it is non-trivial to use the information contained in the Lyman- $\alpha$  forest data in cosmological parameter estimation [61, 62, 63]. Intriguingly, the upper limit on  $m_{\nu, \text{tot}}$  quoted in [1] with the Lyman- $\alpha$  forest data included is actually *weaker* than the one without. The strongest limit to date is, as mentioned in the previous subsection, the one derived by Seljak et al. [56] by combining the power spectrum from the Lyman- $\alpha$  forest with SDSS data on galaxy clustering and galaxy-galaxy lensing and the WMAP CMB power spectrum. Such upper limit  $m_{\nu, \text{tot}} < 0.42$  already gives a non-degenerate mass hierarchy.

### 5.3. Peculiar velocities

The rms bulk flow is predicted in linear theory as:

$$\langle v^2(R_*) \rangle = (2\pi^2)^{-1} H_0^2 f^2(\Omega_m, \Omega_\Lambda) \int dk P(k) W_G^2(kR_*) \quad (8)$$

where  $W_G(kR_*)$  is a window function, e.g.  $W(kR_*) = \exp(-k^2 R_*^2/2)$  for a Gaussian sphere of radius  $R_*$ . The perturbations growth factor is  $f(\Omega_m, \Omega_\Lambda) \approx \Omega_m^{0.6}$ , hence the peculiar velocity field is insensitive to the cosmological constant or dark energy (e.g. [64, 65]). If we parameterise the power spectrum as  $P(k) \equiv Ak^n T^2(k)$  we can then consider two model universes with the same early universe primordial power spectrum  $Ak^n$  but different transfer functions  $T(k)$  according to the contribution of the neutrinos. We define the total mass density parameter as  $\Omega_m = \Omega_c + \Omega_b + \Omega_\nu$  due to the contributions of cold dark matter and neutrinos.

Figure 7 shows the ratio of the bulk flows in two universes with the same primordial power spectra, same  $\Omega_m = 0.3$ ;  $\Omega_b = 0.04$ ;  $h = 0.7$ ;  $n = 1$  but different  $\Omega_\nu = 0$  and  $0.04$ . we can see the suppression of  $\sim 20\%$  of velocities on radii  $R_* < 50h^{-1} \text{ Mpc}$  due to the massive neutrinos.

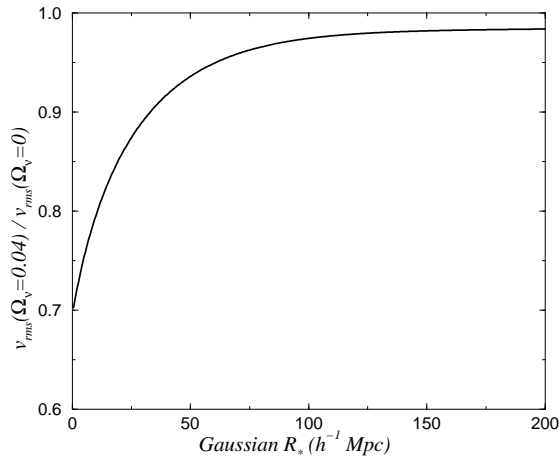
If the power spectrum normalization  $\sigma_8$  is known from another independent measurement, then the amplitude  $A$  itself is inversely proportional to an integral over the power spectrum as

$$\sigma_8^2 = (2\pi^2)^{-1} A \int dk k^2 k^n T^2(k) W_{TH}^2(8k) \quad (9)$$

where  $W_{TH}$  is an  $8 \, h^{-1} \text{ Mpc}$  radius spherical top-hat window function. One can then make predictions for the bulk flows  $v_{rms}/\sigma_8$  in different models, as shown in Figure 8. In part the dramatic differences for models with and without massive neutrinos are due to the  $\sigma_8$  normalization procedure. While it is encouraging that the effect of massive neutrinos is significant for small scales, where peculiar velocities can be measured more accurately to nearby galaxies, we should be aware of complications due to non-linear effects and systematic errors.

### 5.4. Gravitational Lensing

Deep and wide weak lensing surveys will in the future make it possible to perform weak lensing tomography of the matter density field [66, 67]. By binning the galaxies in



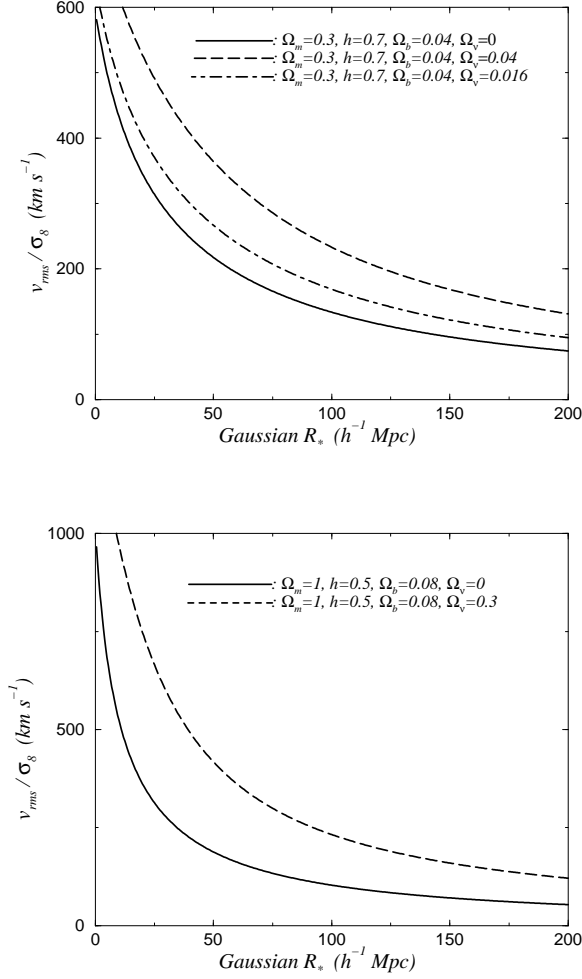
**Figure 7.** The ratio of the bulk flows in Gaussian spheres for two universes with the same primordial power spectra, same  $\Omega_m = 0.3$ ;  $\Omega_b = 0.04$ ;  $h = 0.7$ ;  $n = 1$  but different  $\Omega_\nu = 0$  and  $0.04$ .

a deep and wide survey in redshift, one can probe the evolution of the gravitational potential. However, because massive neutrinos and dark energy have similar effects on this evolution, complementary information is required in order to break this degeneracy. Several studies of the potential of lensing tomography to constrain cosmological parameters, in particular dark energy and neutrino masses, have been carried out, see e.g. [68] for an overview. Even when taking the uncertainties in the properties of dark energy into account, the combination of weak lensing tomography and high-precision CMB experiments may reach sensitivities down to the lower bound of 0.06 eV on the sum of the neutrino masses set by the current oscillation data [68].

## 6. Discussion

The dramatic increase in amount and quality of CMB and large-scale structure data we have seen in cosmology in the last few years have made it possible to derive fairly stringent limits on the neutrino mass scale. Even though the CMB is less sensitive to neutrino masses than large-scale structure data, it plays a crucial role in breaking parameter degeneracies. With the WMAP and SDSS data, the upper limit has been pushed down to  $\sim 1$  eV for the total mass, assuming three massive neutrino species. Results from Lyman- $\alpha$  clouds combined with the WMAP and SDSS galaxy clustering data provide a tighter upper limit of  $\sim 0.4$  eV.

One point to bear in mind is that all these limits assume the ‘concordance’  $\Lambda$ CDM model with adiabatic, scale-free primordial fluctuations. While the wealth of cosmological data strongly indicate that this is the consistent basic picture, one should keep in mind that cosmological neutrino mass limits are model-dependent, and that



**Figure 8.** The effect of massive neutrinos on peculiar velocities in  $\Lambda$ CDM models (top panel) and MDM models (bottom panel).

there might still be surprises. For example, as the suppression of the power spectrum depends on the ratio  $\Omega_\nu/\Omega_m$ , we found in [51] that the out-of-fashion Mixed Dark Matter (MDM) model, with  $\Omega_\nu = 0.2$ ,  $\Omega_m = 1$  and no cosmological constant, fits the 2dFGRS power spectrum well, but only for a Hubble constant  $H_0 < 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . A similar conclusion was reached in [69], and they also found that the CMB power spectrum could be fitted well by the same MDM model if one allows features in the primordial power spectrum. Another consequence of this is that excluding low values of the Hubble constant, e.g. with the HST Key Project, is important in order to get a strong upper limit on the neutrino masses.

If the future observations live up to their promise, the prospects for pushing the cosmological neutrino mass limit down towards 0.1 eV are good. Then, as pointed out in [70], one may even start to see effects of the different mass hierarchies (normal or inverted), and thus one should take this into account when calculating CMB and

matter power spectra. For example, with a non-degenerate mass hierarchy one will get more than one free-streaming scale, and this will leave an imprint on the matter power spectrum. The coming years will see further comparison between the effective neutrino mass in Tritium beta decay, the effective Majorana neutrino mass in neutrinoless double beta decay and the sum of the neutrino masses from cosmology [71], [72]. It would be a great triumph for cosmology if the neutrino mass hierarchy were finally revealed by the distribution of large-scale structures in the Universe.

## Acknowledgments

We thank S. Bridle and M. Fukugita for valuable discussions. ØE acknowledges support from the Research Council of Norway through grant number 159637/V30 and OL thanks PPARC for a Senior Research Fellowship.

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